**ABSTRACT:**

These notes apply and extend our Crowd Physics analysis (previously successful in modeling “milling behavior” in collections of fruit flies) to the case of time-dependent migration flows. The underlying model and its rationale are largely the same, the analysis is a bit different and builds on previously theoretical work on “currents” or “flows” in non-equilibrium systems.

**Key concepts of underlying model (description adapted to human migration context):**

It is commonly observed in physics that, for systems of large numbers of particles, emergent systems behavior are quite general and independent of the exact details of the interactions between particles. Accordingly, we construct the simplest possible underlying model capable of quantifying the key influences on the system, and then apply mathematical analyses appropriate for large numbers of individuals in order to draw out quantitative predictions of the relationships between those external influences and the behavior of the system.

Specifically, in this case, we consider an economics-like utility-optimization view of the behavior of households considering potentials moves to new Prefectures. We take each household *h* to reside in a given prefecture *ph*. To quantify the effects of the relative attractiveness of each prefecture *p* (corresponding to economic opportunity, perceptions of safety, etc.) we assign to each an overall utility value *V*(*p*). Moreover, to include the effects of changes in the population density *n*(*p*) each prefecture (affecting for example the availability and affordability of housing), we introduce a second, density-dependent utility term *f’*(*n*(*p*)) for each prefecture *p*, so that the quantity which each household actually seeks to optimize is *V*(*p*)+*f’*(*n*(*p*)). (Our motivation for writing the derivative of *f*(*n*) instead of the function itself will be apparent in our exact solution below.) We note that we have demonstrated in living systems that these utility functions are not arbitrary but can be extracted directly from data on population fluctuations when the system is in a near steady state (when, for example, there has been no sudden disaster).

Finally, we take households at prefecture *p* to migrate to other prefectures *p’* at a rate related to the change in the value of *V*(*p*)+*f’*(*n*(*p*)) in make the transition. Calling this change *ΔH*, we take the probability of migrating to prefecture *p’* during a given time interval to be exp(*ΔH*)/(exp(*ΔH*)+1)/(*N*-1), where *N*

quantities *V*(*p*) and *f*(*n*)

Note that, here and throughout, we follow the Physics (rather than Economics) convention, in which agents seek to *minimize* (rather than maximize) utilities. Correspondingly, we refer to the “vexation” *V*(*x*) and “frustration” *f*(*n*), as quantities the agents seek to minimize, with *V*(*x*) playing a role analogous to an external potential-energy function.

*Exact solution to deterministic model:* Under conditions in which agents strictly seek to optimize their utility, agents will adjust their locations until no location *x* has a lower net utility *V*(*x*)+*f’*(*n*(*x*)) than any other, at which point all motion will cease and a static equilibrium is established. At this point, the population is distributed so that the utility function for all locations has taken on a constant value,

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|  | *μ* = *V*(*x*) + *f’*(*n*(*x*)) . | (1) |

To interpret Eq. (1), we note that this condition corresponds precisely to minimization of the following global crowd-utility function *H* under the constraint of fixed number of agents *N* ≡ ∫ *n*(*x*) *dA,*

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|  | *H* = *F*[*n*(*x*)] + ∫ *V*(*x*) *n*(*x*) *dA*, | (2) |

where *μ* is the Lagrange-multiplier maintaining the fixed number of agents, and *F*[*n*(*x*)] ≡ ∫ *f*(*n*(*x*)) *dA* is a functional (a map from an entire function *n*(*x*) to a single real number) of the population density alone, regardless of the external influences as characterized by *V*(*x*).[[1]](#footnote-0) From the role of *μ* in Eqs. (1,2), it is clear that *μ* is directly analogous to the chemical potential from statistical physics, which takes on a constant value across a system in which particles have freedom of movement.

1. We write Eq. (2) using a generic functional form *F*[*n*(*x*)] to allow also for the possibility of more general frustration effects at each point *x* that may depend on more than simply the local density *n*(*x*). The particular form corresponding to Eq. (1) is known as a “local density approximation” (LDA) in the density functional theory literature [KohnSham]. [↑](#footnote-ref-0)